DUAL MODE CONTROL

REPORT

Submitted in Partial Fulfillment
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by

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ABSTRACT

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by

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Adviser: Robert Staffin

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The purpose of this report is to achieve a simple compensation scheme for the control of a process. The criteria are that the process output closely approximate the process input when the latter is a step and that there be no steady state error. The reference process chosen is that of chemical concentration control.

Employing a compensation block with pure gain gives a fast response that has a steady-state error. Integral compensation eliminates the steady-state error, but its response has a very sluggish transient character. A switching arrangement is evolved that combines the desirable characteristics of both types of compensations while removing the unwanted traits.

This configuration is set up on an analog computer and compound results which satisfy the original criteria are empiracally obtained for a plant of three poles. This is done for plants of two poles and one pole with equal success, exhibiting the validity of the technique for a general plant.

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I. Consideration of An Actual Process:

The example chosen is that of a chemical processing problem involving concentration control. The object is to describe a physical system and evolve its transfer function so that the subject matter covered in this report is immediately applicable to a real situation.

The main component in the concentration control problem is the mixing tank:
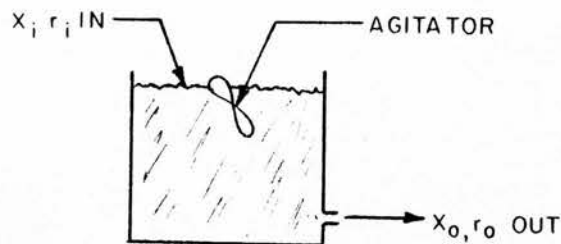
Fig. 1 - Mixing Tank

The input to tank (X_i) is given in pounds of dissolved substance per gallon of mixture. The mixture is allowed to flow into the tank at the rate of r_i gallons per minute. The concentration uniformity is maintained by agitators. The output of the tank is similarly measured in pounds of substance per gallon of mixture, flowing out at the rate of r_o gallons per minute.

Since the amount of substance added to the tank (ΔQ) during any time interval is the amount entering minus the amount leaving over that same time interval, ΔQ is given by

$$\Delta Q = x_i r_i \, \Delta t - x_o r_o \, \Delta t, \text{ assuming } x_i, x_o, r_i, r_o, \text{ are constant}$$
$$\text{over } \Delta t. \tag{1}$$

then,

$$\frac{\Delta Q(t)}{\Delta t} = x_i(t) \, r_i(t) - x_o(t) \, r_o(t). \tag{2}$$

Taking the $\lim\limits_{\Delta t \to o} \dfrac{\Delta Q(t)}{\Delta t}$, $\dfrac{dQ(t)}{dt}$ is obtained. This is the instantaneous rate of change of the amount of substance in the tank at any time t. But, $x_o(t)$ is the output concentration or simply the amount of substance $Q(t)$ in the tank at any time t divided by the volume $(v(t))$ of mixture in the tank.

Therefore

$$\frac{dQ(t)}{dt} = x_i(t)\, r_i(t) - \frac{Q(t)}{v(t)}\, r_o(t) \tag{3}$$

Transposing,

$$\frac{dQ(t)}{dt} + \frac{Q(t)}{v(t)}\, r_o(t) = x_i(t)\, r_i(t). \tag{4}$$

If the rates in and out are made constant and equal, the volume B is also a constant. This results in

$$\frac{dQ(t)}{dt} + a\, Q(t) = r_{i_1} x_i(t), \quad \text{where } a = \frac{r_{o_1}}{v_1}, \; r_{o_1}, \; r_{i_1}, \; v_1 \text{ are}$$

$$\text{constants.} \tag{5}$$

The La Place Transform of equation 5 is

$$Q(s) \left[\, s + a \,\right] = r_{i_1} X_i(s) \,. \tag{6}$$

Or,

$$\frac{Q(s)}{X_i(s)} = \frac{r_{i_1}}{s+a} \,. \tag{7}$$

Equation 7 is the transfer function for a perfectly mixed vessel. Thus, the mixing tank can be represented as follows (in block diagram form):
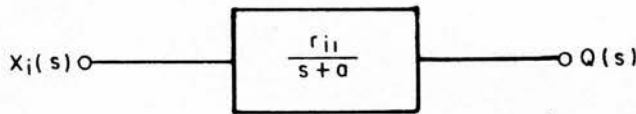

Fig. 2 - Block Diagram of Mixing Tank

Since $X_o(s) = \dfrac{Q(s)}{v_1}$, two identical tanks in series can be represented as:
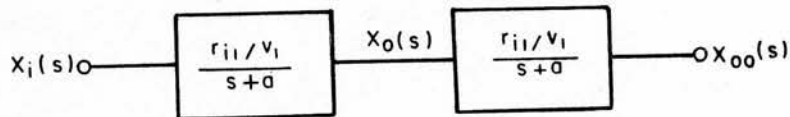

Fig. 3 - Block Diagram of two Series Mixing Tanks

To obtain a general pneumatic valve transfer function, consider the diagram of figure 4.
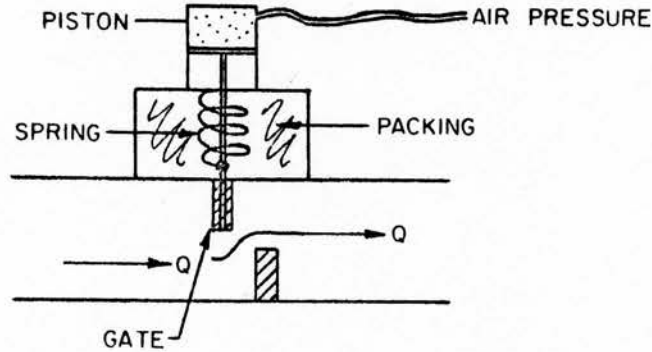
Fig. 4 - Pneumatic Value

Air pressure holds a piston against a restoring spring, thus positioning a gate which determines the flow rate. Summing the forces acting on the value for a given vertical displacement (y), one obtains:

$$M\frac{d^2y}{dt^2} + F\frac{dy}{dt} + Ky = P(t)A,$$ where $P(t)$ is the pressure on the (8) piston as a function of time, A is the area of the piston head, K is the spring constant, F is the viscous damping (due principally to the packing of the gate shaft) and M is the mass of the gate and shaft assembly.

Normally, the $F\frac{dy}{df}$ term is much larger than the $M\frac{d^2y}{dt^2}$ term, partly because the damping coefficient F is larger than the mass M and partly because the acceleration is quite low. These considerations allow the following approximation:

$$F\frac{dy}{dt} + Ky = P(t)A. \qquad (9)$$

or,

$$\frac{F}{K}\frac{dy}{dt} - Y = \frac{A}{K}P(t),$$ where $\frac{A}{K}$ is called the valve constant and

$$\frac{F}{K}$$ the valve time constant. (10)

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The La Place Transform of Equation 10 yields:

$$Y(s) + \left(\frac{F}{K} S + 1 \right) = \frac{A}{K} P(s) \tag{11}$$

$$\frac{Y(s)}{P(s)} = \frac{A/K}{1 + S \, F/K} . \tag{12}$$

Usually, the output of the control valve is fed into a mixing valve. Here the substance and perhaps a solvent are combined. Assuming that the amount of substance per gallon of mixture is small or that the rate of mixture flow is independently held constant, then the control valve position is seen to directly determine the concentration of the mixing valve combination:

$$\frac{X_i(s)}{P(s)} = \frac{A'}{1 + S(F/K)} , \tag{13}$$

where $X_i(s)$ is the concentration of the mixture and A' is a compound constant.

Consider a system composed of a control valve, a mixing valve, and two identical tanks arranged as shown in figure 5:
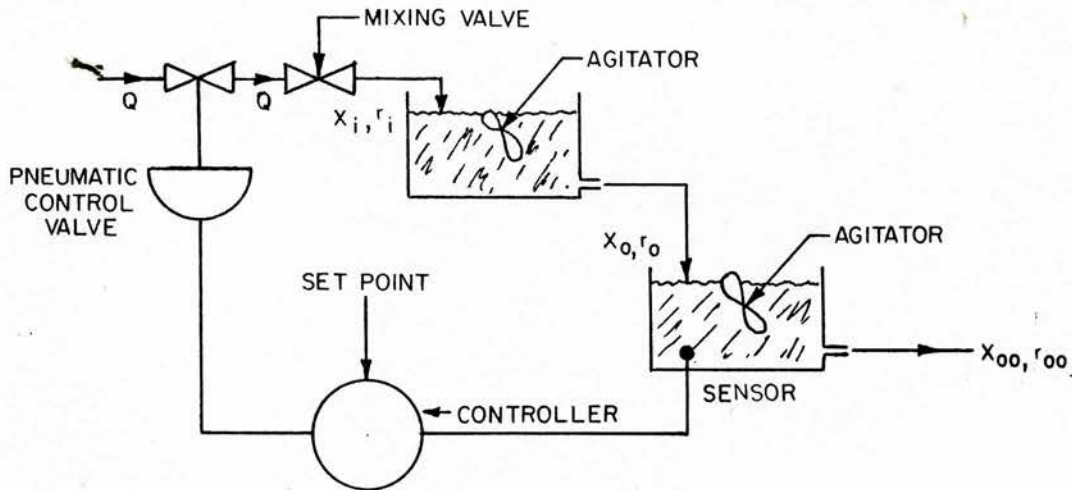

Fig. 5 - Chemical System

The configuration shows the inclusion of a return loop as a means of automatically controlling the concentration. The path consists of a sensing element, a controller - whose transfer function is given by

$$G(s) = Ks + K' + \frac{K''}{S} , \tag{14}$$

and an output which varies about a reference concentration input (called the set point).

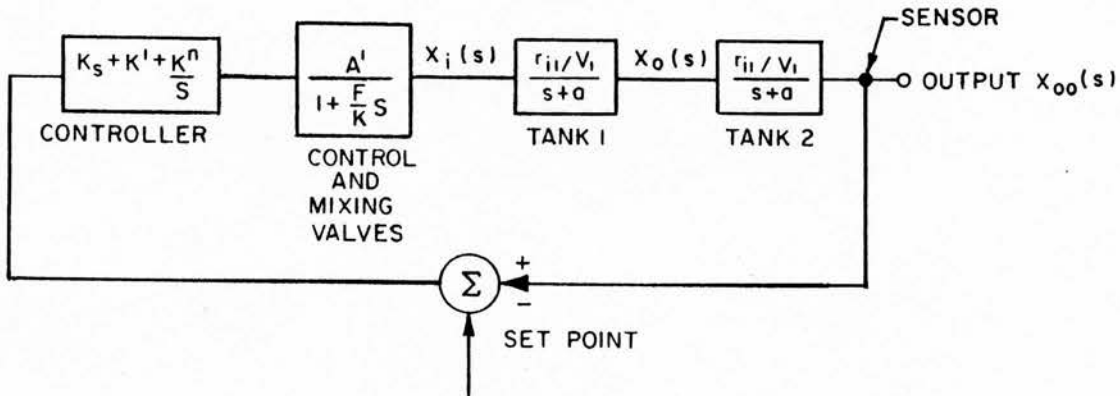Representing the chemical system in block diagram form:

Fig. 6 - Chemical System in Block Diagram Form

II. Statement of the Problem:

Observing that the input to the system is the set point, figure 6 is simplified by selecting values for the constants and combining the blocks of the chemical process:
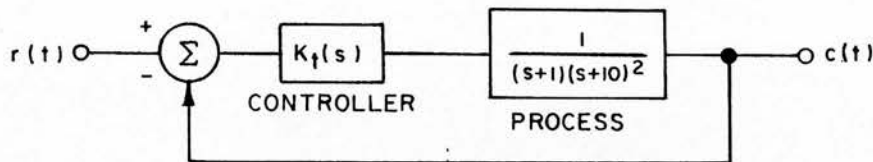
Fig. 7 - Generalized System

Focusing attention on the return loop, it is emphasized that feedback is an excellent method for controlling a system process. Normally, the process can be represented by its frequency domain poles and zeroes together with a scale factor. The basic behavior of a system is determined by the roots of its transfer function denominator. These poles form the fundamental terms for a partial fraction expansion. Once the expansion is known in the frequency domain, the time domain behavior is specified for any given input

via the inverse La Place Transform.

The controlling block incorporated in the forward path usually contains a variable gain. Changing the value of this gain produces corresponding variations in the location of the overall system's pole-zero locations. This in turn changes the characterization of the system performance. For given process poles and zeroes, the Root Locus Technique is a useful tool for establishing a relationship between closed loop singularity positions and variation in forward path gain.

Assuming that the controller is a variable gain only and sketching the resulting pole-zero pattern (see fig. 8), a few desirable step responses are then calculated for particular values of gain (see fig. 9). These responses would be adequate, were it not desired that the output should exactly follow the input in steady state. This cannot occur with this system.

The overall transfer function is:

$$\frac{c(t)}{r(t)} = \frac{k_1}{k_1 + (s+1)(s+10)^2} \qquad \text{(15)}$$

now, applying the final value theorem (with the input a unit step):

$$c(t = \infty) = \lim_{s \to 0} \; s\left[\frac{k_1}{k_1+(s+1)s+10)^2}\right] \frac{1}{s} \qquad \text{(16)}$$

$$= \frac{k_1}{100 + k_1} \; . \quad \text{This cannot be 1 for a stable system.} \qquad \text{(17)}$$

This leads to the consideration of a controller transfer function with a variable gain multiplied by an integrating $\frac{1}{S}$ term. The overall transfer function here is:

$$\frac{c(t)}{r(t)} = \frac{k_2}{k_2+s(s+1)(s+10)^2} \; . \quad \text{Applying the final value theorem} \qquad \text{(18)}$$

again (the input still a unit step):

$$c(t = \infty) = \lim_{s \to 0} \; s\left[\frac{k_2}{k_2+s(s+1)(s+10)^2}\right] \frac{1}{s} \qquad \text{(19)}$$

$$= 1 \; . \quad \text{This is unity independent of } k_2 \; . \qquad \text{(20)}$$

If the criterion for the system is merely that it have no error in steady state response to a step drive, the integrator with variable gain serves nicely.
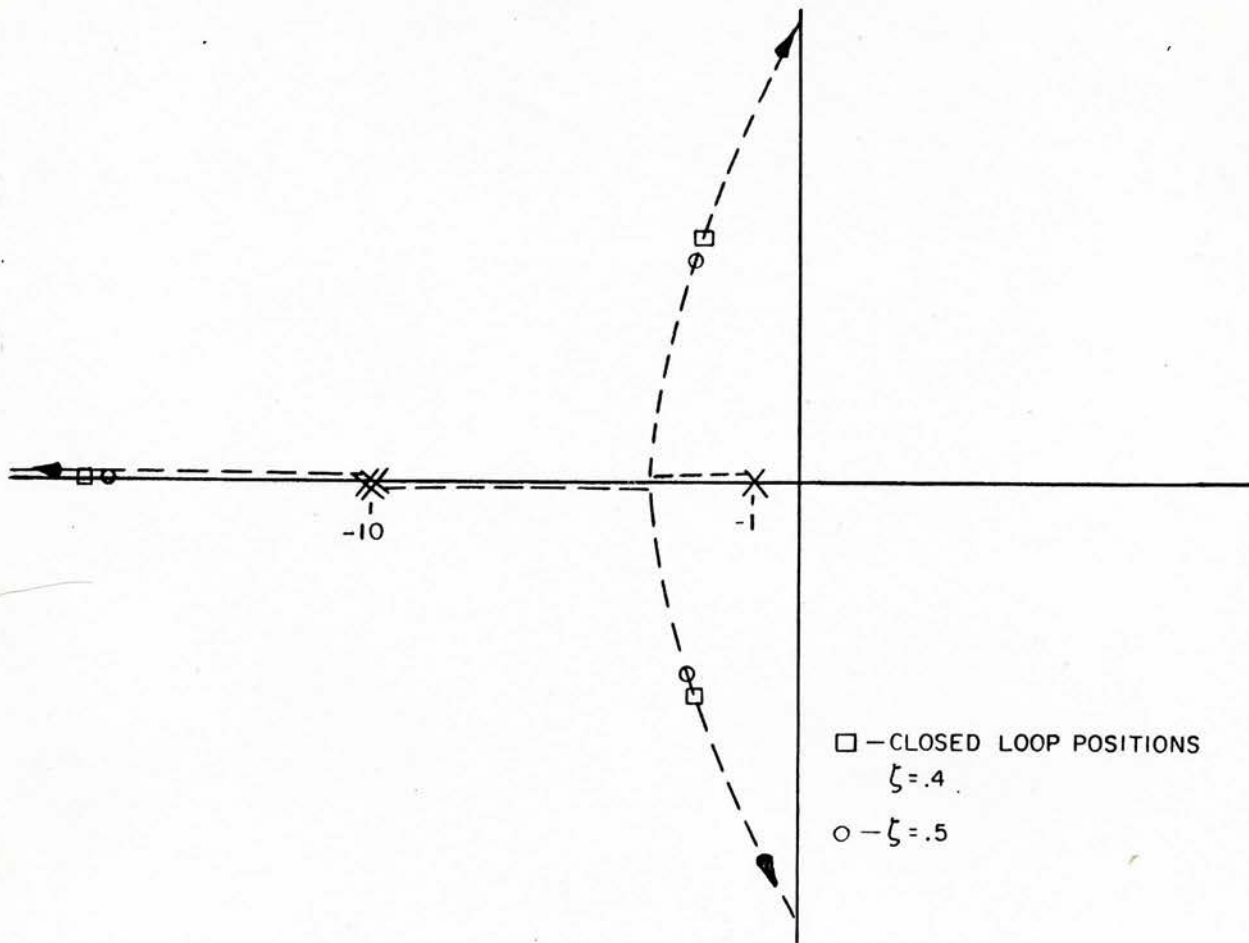
Fig. 8 - Mode 1 Pole-Zero Plot

□ —CLOSED LOOP POSITIONS
$\zeta = .4$

o — $\zeta = .5$

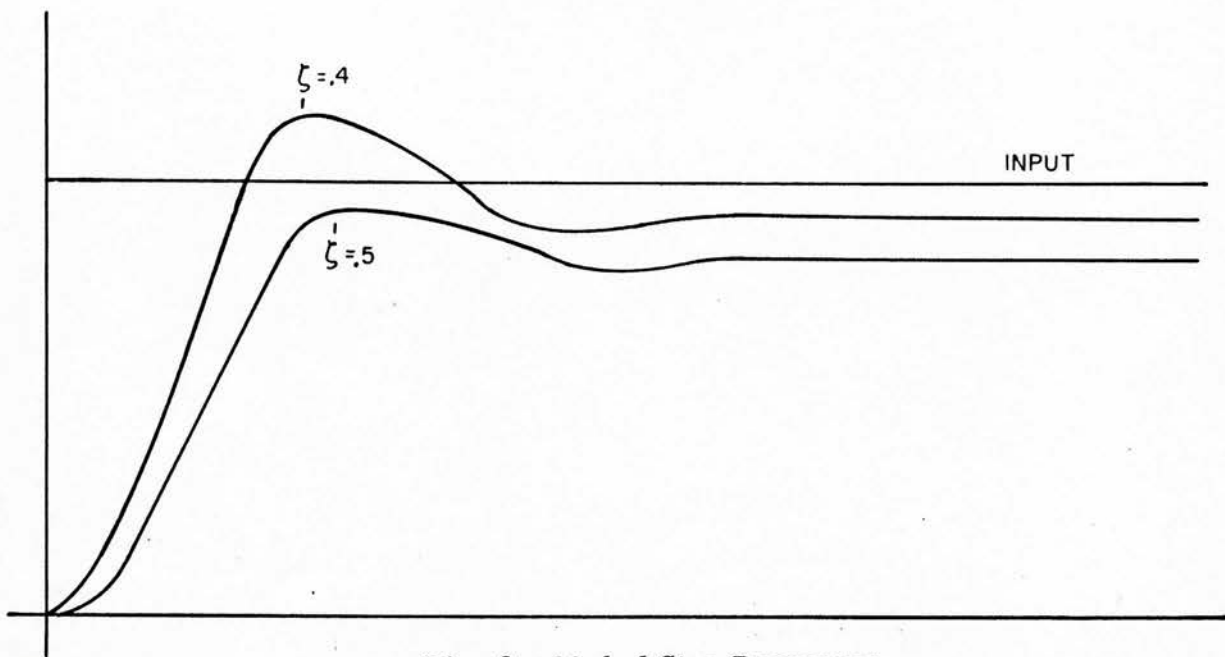

$\zeta = .4$

INPUT

$\zeta = .5$

Fig. 9 - Mode 1 Step Responses

However, if the constraint of a speedy transient response is added, this
system is no longer satisfactory.

From the pole-zero pattern of the system with integral compensation
(see Fig. 10), it is seen that the system behaves as a dominant second
order system. This is so since both the time constants and the residues
associated with the pole pair deep in the left half plane are much smaller
than those of the pair of poles closest to the origin. With the system de-
signed for a moderate peak in step response, the time constant of the
dominant poles is seen to be on the order of 3 seconds. Looking back to
the straight gain compensation scheme, it too is seen to approximate a
second order system. However its transient time constants are seen to
be approximately .5 seconds. That the difference between 3 and .5 is
significant is brought out by the fact that time constants expressed in hours
are not unusual when dealing with chemical systems. In the generalized
block diagram under consideration, all time constants are normalized to
be given in seconds.

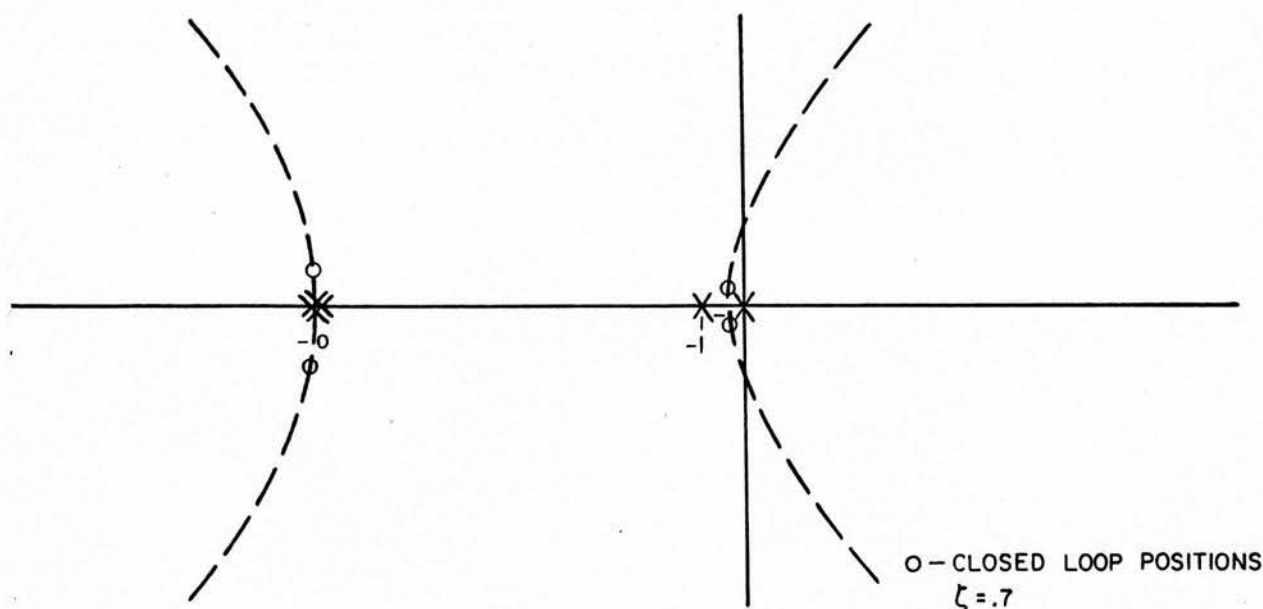O – CLOSED LOOP POSITIONS
$\zeta = .7$

Fig. 10 - Mode 2 Pole - Zero plot

With this motivation in mind, a switching arrangement is envisaged in an attempt to gain the desirable characteristics of both controller block settings while eliminating their unattractive aspects:

$$r(t) \xrightarrow{+ \atop -} \Sigma \longrightarrow \begin{cases} K_1 \\ \dfrac{K_2}{S} \end{cases} \xrightarrow{\;1\;} \boxed{\dfrac{1}{(s+1)(s+10)^2}} \longrightarrow c(t)$$
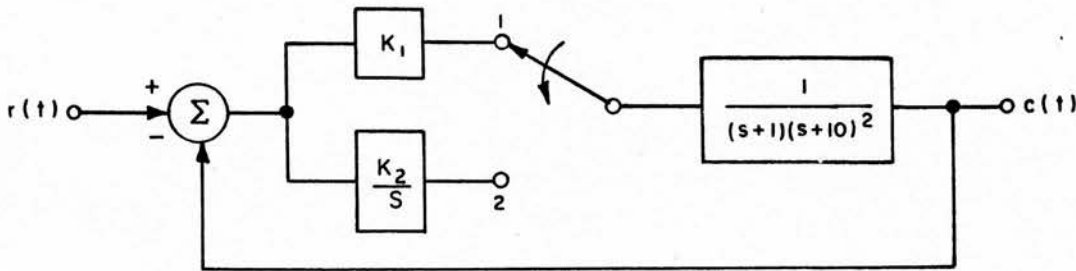
Fig. 11. - Switching arrangement

At this point it is necessary to note that a fast errorless system can be achieved thru the use of ordinary, existing compensation methods. The approach offered here however, has the advantages of simplicity, low cost, and versatility while remaining extendable to non-linear devices.

III. Experimental Solution of the Problem: (A.)

The overal system is to be designed as combinations of switch position 1 (mode 1) and switch position 2 (mode 2). This investigation is most easily carried out by instrumenting the entire problem on an analog computer.

Regarding the block diagram of Fig. 11, one of the first difficulties that arrises is that a step discontinuity in value will occur at the wiper of the switch when it is changed from position 1 to position 2. This effective step input at the time of switching occurs because the output of the straight gain is not the same value as the output of the integrator. This difficulty is circumvented in the instrumentation of the compensation block on the analog computer (Fig. 12 is the entire program). Here R" and C" are chosen so that their parallel combination approximates a pure gain, while switching out R" and R_2 obviously yields the integrator with its own gain value. It is noted that conversion from one to the other yields no discontinuity in voltage appearing across the capacitor. However, discontinuities in derivatives of the voltage remain as a possibility.
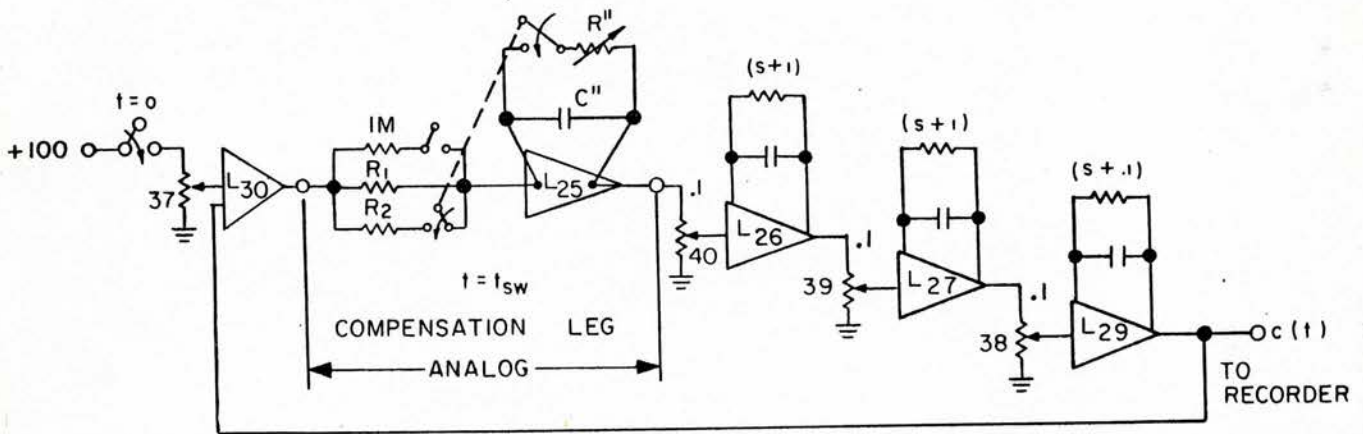
Fig. 12 - Analog Computer set-up

NOTE: Time constants have been multiplied by 10 so as to be commensurate with the recording devices used.

For the R" C" combination, the transfer function is $\dfrac{-\dfrac{1}{R_2 C"}}{S + \dfrac{1}{R" C"}}$. For

$S << \dfrac{1}{R" C"}$ this becomes $\dfrac{R"}{R_2}$. Alternately, to force this to follow a step quickly, it is desired that $\dfrac{1}{R" C"}$ be as large as possible. The value of $\dfrac{1}{R" C"}$ limits the approximation of a pure gain. In this instrumentation $\dfrac{1}{R" C"}$ is limited to 40.

The time for switching is determined by visually following the output as displayed on a recorder and throwing a toggle switch when a desired level is reached. After executing a few runs with randomly selected switching points, a pattern becomes evident; switching early produces large overshoots while switching late yields "under shoots". A few more experimental attempts with the above pattern as a guide quickly yields intermediate responses which do incorporate the best features of both systems; those of being fast, errorless, and close approximations of the input drives (these results are summarized in Fig. 13).
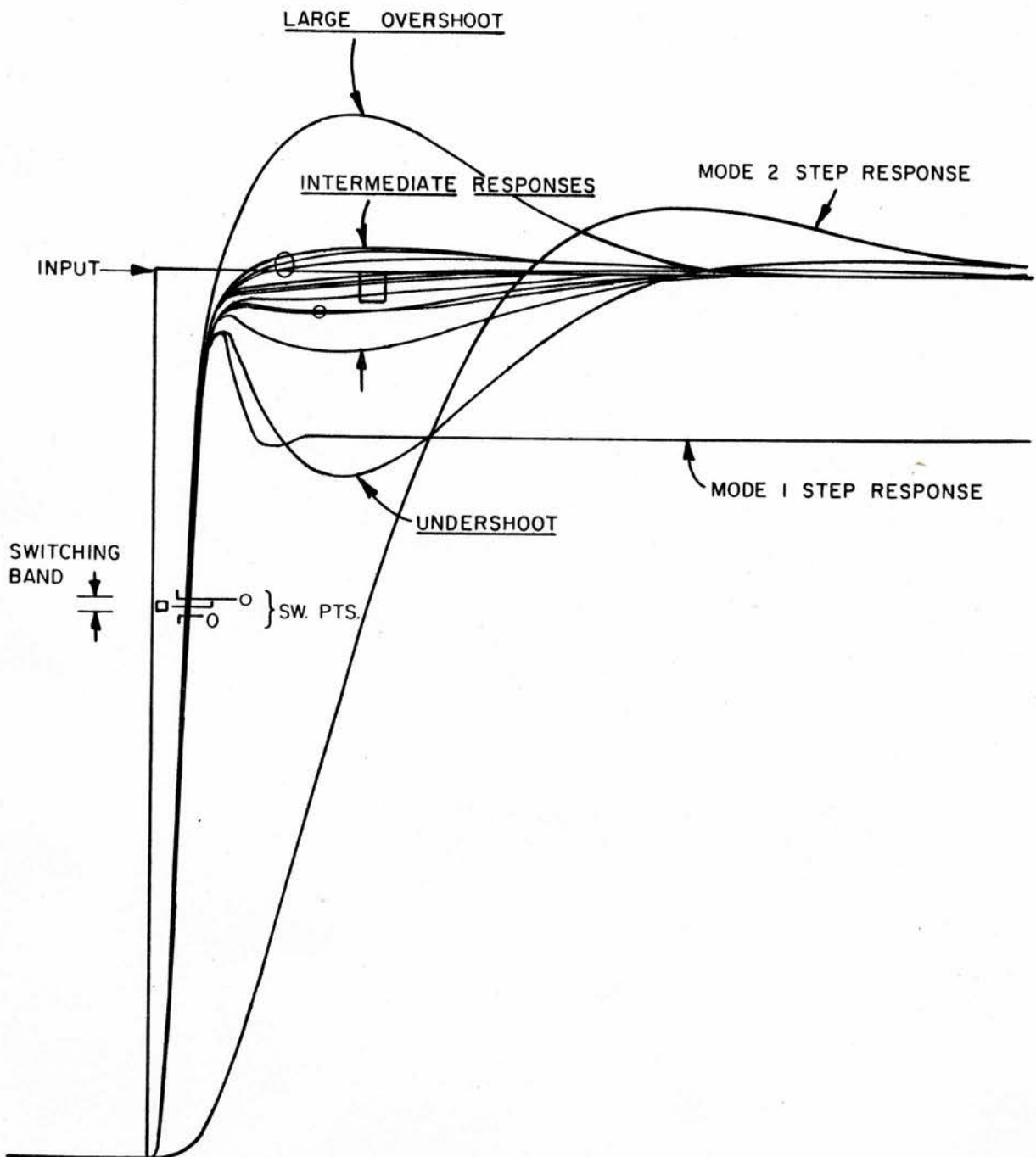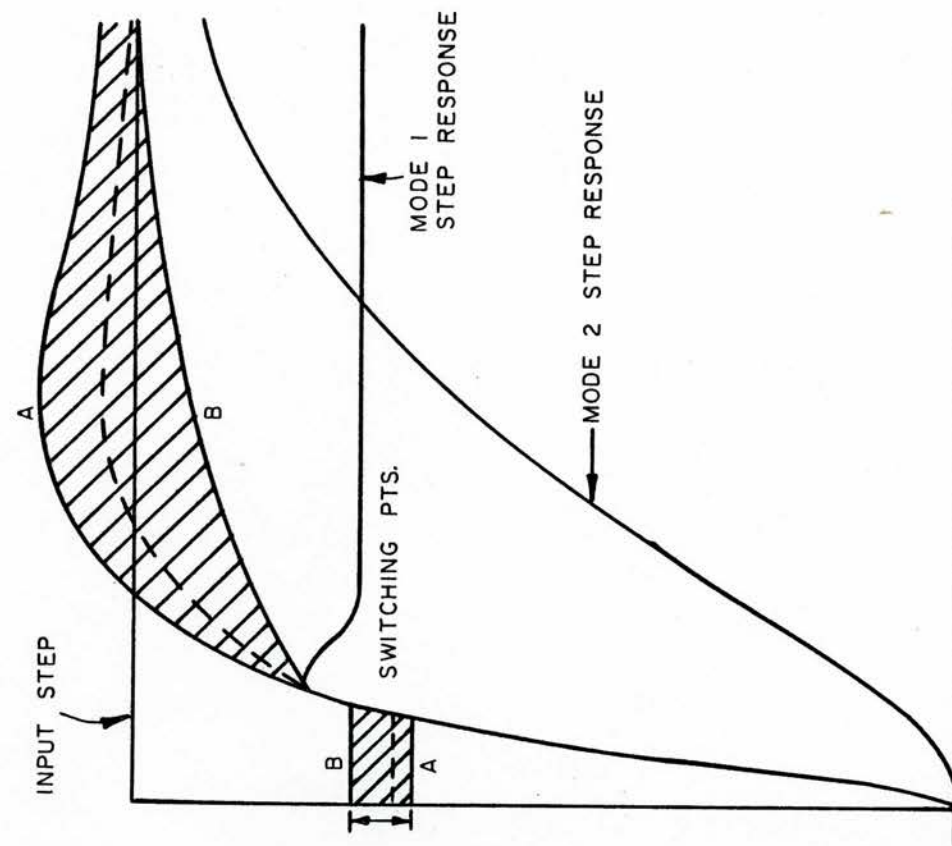
Fig. 13 - Empirical Responses
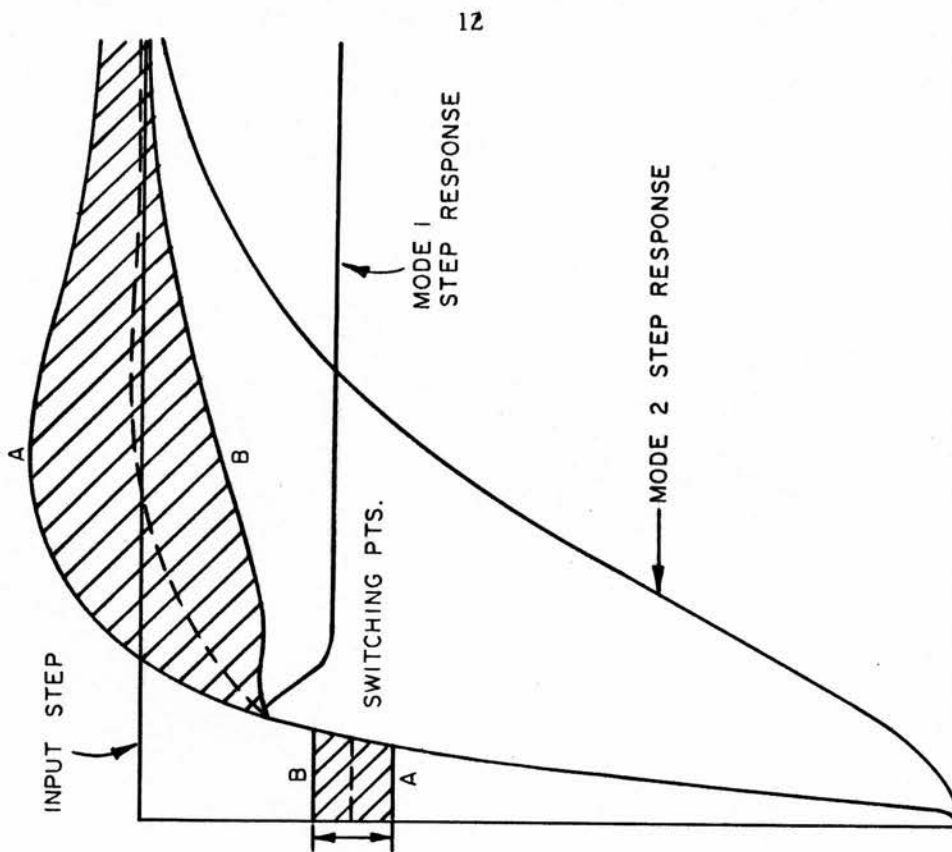
Fig. 14 - Responses with Pot 38 at .25

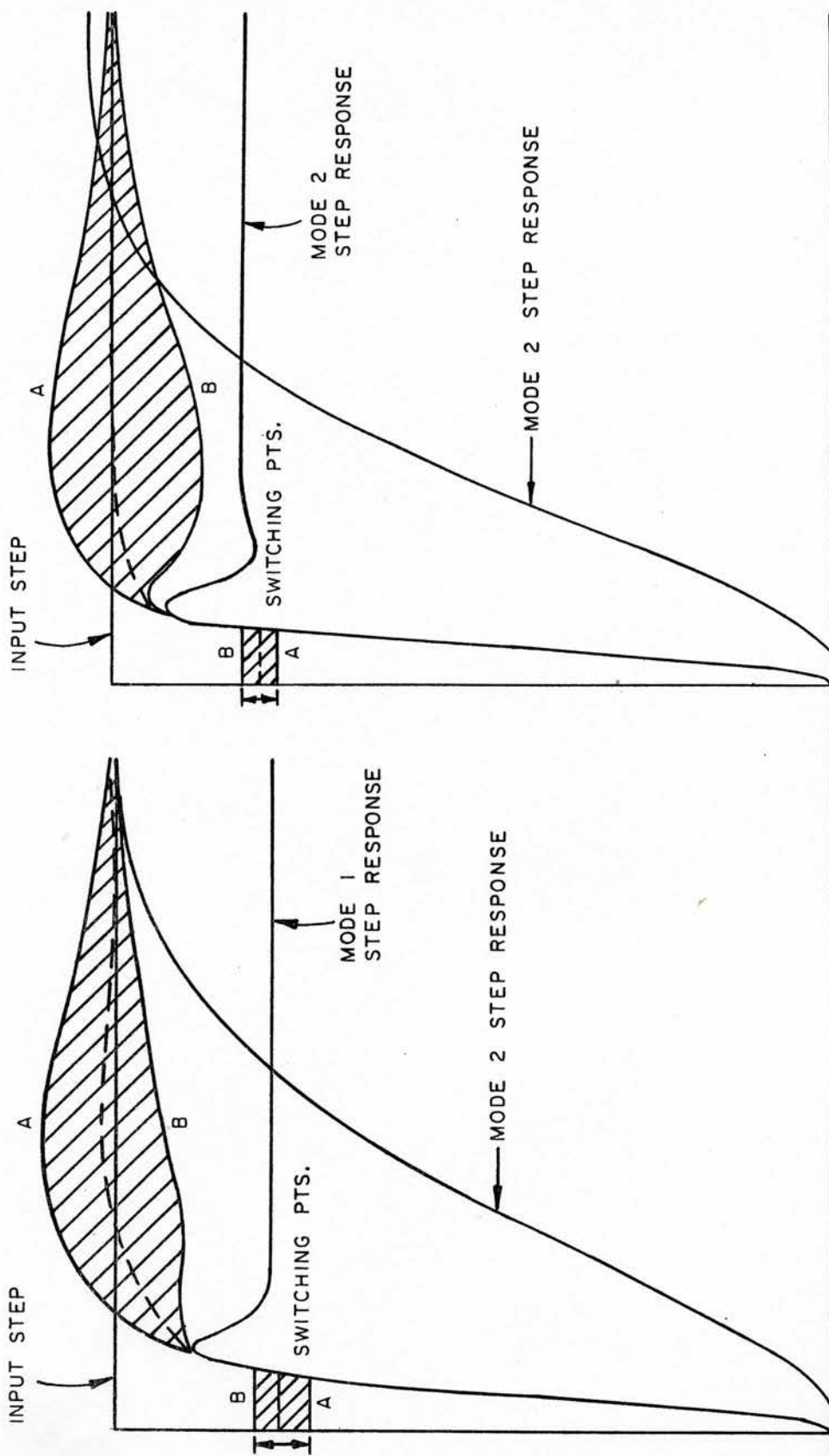Fig. 15 - Responses with Pot 38 at .30

Fig. 16 - Responses with Pot 38 at .35
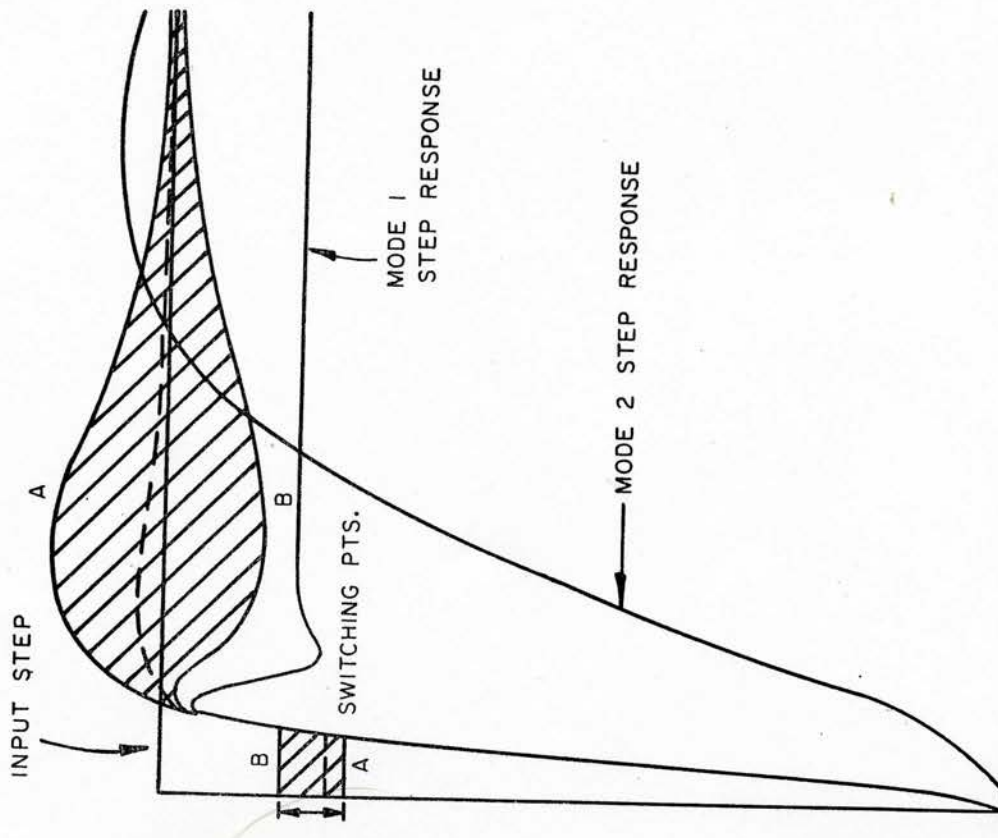
Fig. 17 - Responses with Pot 38 at .42

Fig. 18 - Responses with Pot 38 at .475
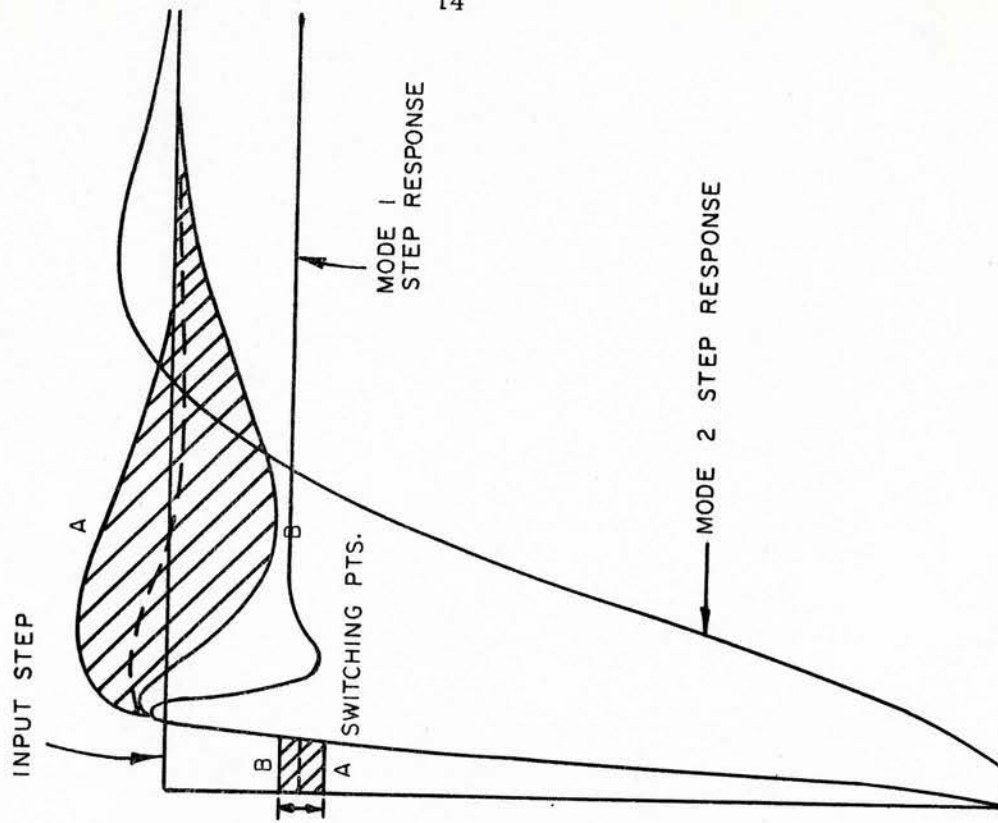
Fig. 19 - Responses with Pot 38 at .55

NOTE: APPROXIMATION LIMITATIONS PRECLUDE GROSS VARIATIONS IN R" (SEE FIG. 12) THUS IT IS MORE PRACTICAL TO VARY POT 38 (AND THUS THE FORWARD GAIN) TO PRODUCE DESIRED FIRST SYSTEM PEAK RESPONSES. UNFORTUNATELY CHANGING POT 38 PRODUCES A GAIN CHANGE FOR THE SECOND SYSTEM ALSO. THUS, IN THE PREVIOUS PLOTS THE PEAK HEIGHT OF THE SECOND SYSTEM IS SEEN TO CHANGE ACCORDING TO THE SETTING OF POT 38. IT IS DESIRED TO HAVE THE SECOND SYSTEM RESPOND TO A STEP DRIVE WITH A SMALL PEAK OVERSHOOT. THIS MINIMIZES SECOND SYSTEM TIME CONSTANTS AND OSCILLATIONS.

INPUT STEP

MODE 1 STEP RESPONSE

MODE 2 STEP RESPONSE

A SWITCHING PTS.

B

Fig. 20 - Responses with Pot 38 at .62

Performing similar series of runs for different mode 1 peak heights corroborates the results given above. A best switching point and resultant response is readi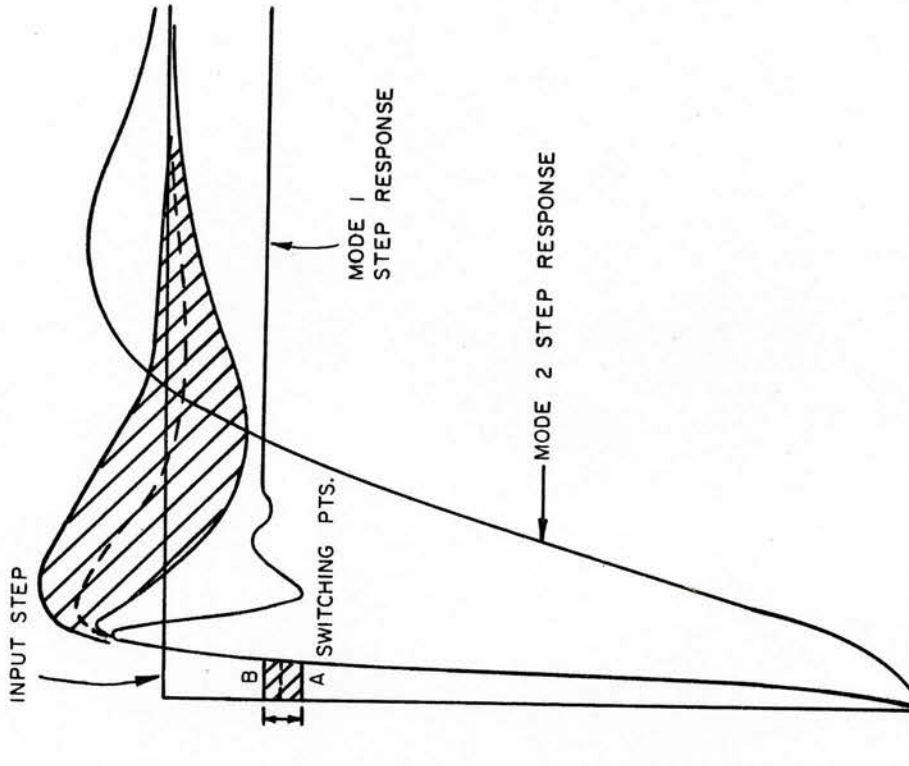ly obtainable by trial and error for each setting of the mode 1 peak height. These curves are given in figures 14 thru 20. In addition to the input and the response of both modes, two arbitrary bounding responses, together with a best response (dashed line) is indicated. The area between the bounding responses is shaded to underline the fact that these responses form a continuous, monotonic, non-intersecting spectrum of responses for monotonic changes in switching times. The fact that the overall system converges rapidly (experimentally) to a suitable response is underscored by the unsophisticated switching criteria and techniques used. Finally, the optimal responses are obtained when the peak for mode 1 occurs near (and especially below) the reference input. This is so since it is observed that the overall response is coincident with mode 1's step response (almost until the latter's peak). Afterwards it either continues upward, flattens out just above the first system's peak, or heads back down. For a step-like output, a sharp corner is desired. Therefore, a mode 1 peak is designed which will allow the compound response to flatten out at its steady state value. Obviously, a mode 1 peak just below the steady state reference is desirable (see figures 17 and 18). Raising the peak much above reference will force the overall response to have a large peak because of the "coincident" property (see figures 19 and 20). Lowering the peak a good deal below reference causes the overall output to lose its corner, forcing it to become greatly rounded (see figures 14, 15 and 16).

B. Investigating the generality of the techniques, a plant with two poles is considered.

Fig. 21 - Block Diagram and Computer Set-up for Two Poles

Sketching the pole-zero plots for the two systems:

Fig. 22 - Mode 1 and Mode 2 Pole-Zero Plots for Two Poles

The statements made and the plots observed for the overall system involving a plant with three poles are found to be characteristic also of a plant with two poles. This is easily seen from a typical family of curves for a particular first and second system (both adjusted to behave as dominant second order systems):
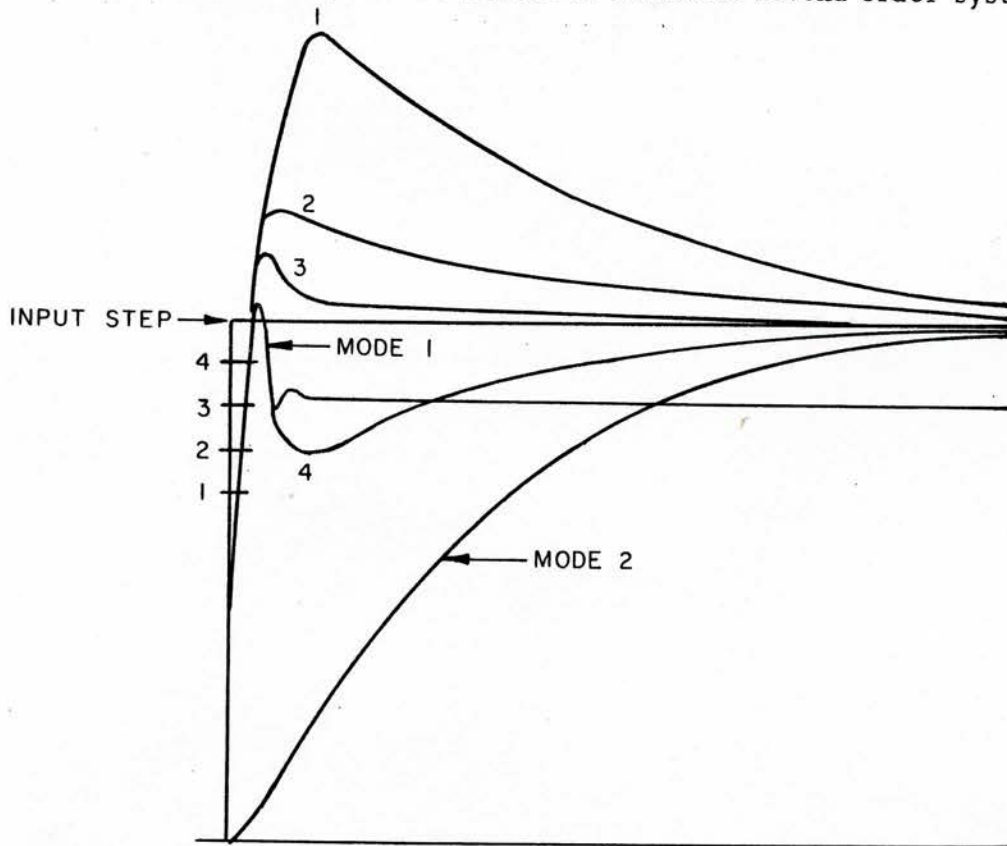
Fig. 23 - Responses For Two Poles

C.   Abandoning the constraint of dominant second order individual system be-
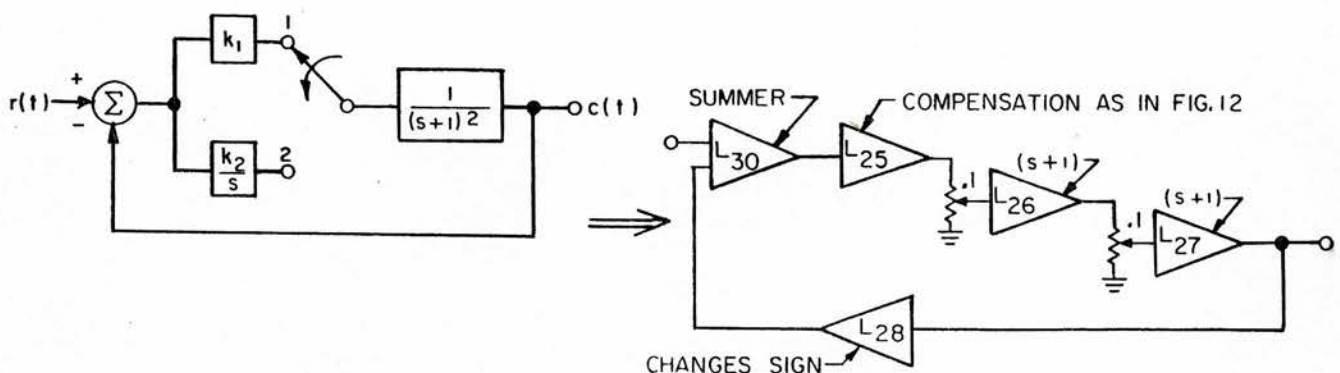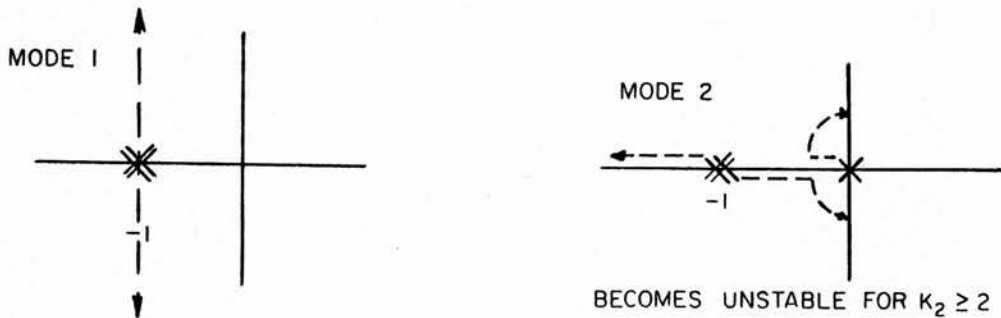havior, a plant with a simple pole is approached:
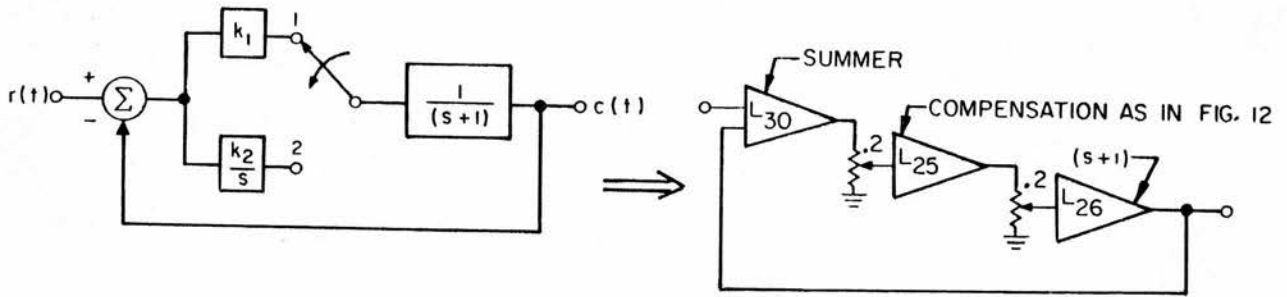
Fig. 24 - Block Diagram and Computer Set-up for One Pole

Sketching the pole-zero plots for the individual system:

Fig. 25 - Mode 1 and Mode 2 Pole-Zero Plots for One Pole

Here again the characterization previously given to the overall systems
continues to be evident. Similar switching criteria can quickly be experimentally
determined to yield overall responses that are alike in nature to those gotten
with the previous plants (of course the first system cannot peak higher than the
input step as was possible before and is step-like in shape). A sample run suffices
to show the applicability of the technique in this situation:
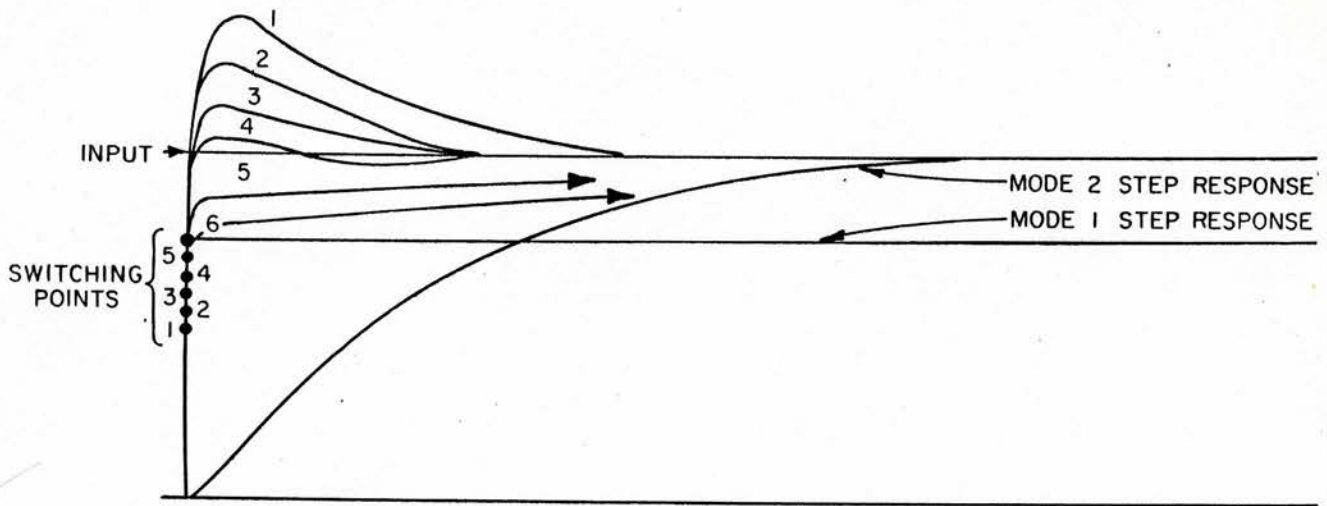
Fig. 26 - Responses for One Pole

Thus, the technique appears to be applicable to a great many plants (which can be thought of as being combinations of the plants considered here).

IV.   Permutation of the Configuration:

It is of interest to note that a permutation of plant and compensation relative to the previous forward path produces an overall response of entirely different characteristics.   This statement holds for all the plants previously investigated. Though the plant is no longer the system output and is therefore not the object of control any longer, it is desired to justify the nature of this new response.   The following configuration yields the set of curves illustrated in fig. 28:
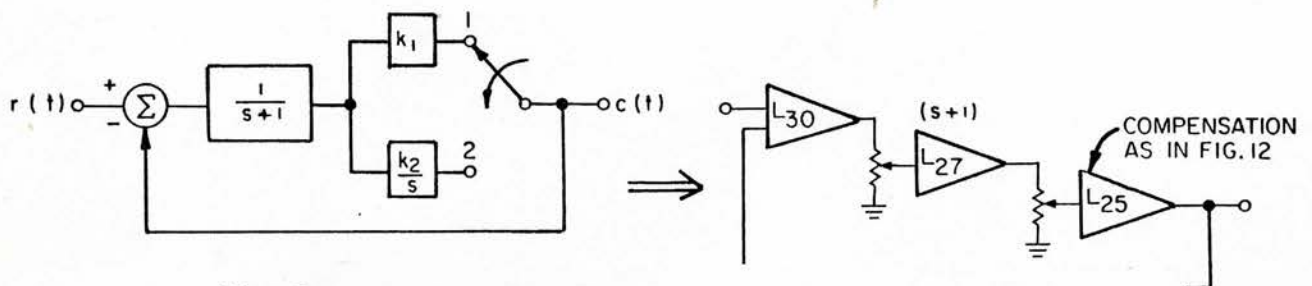
Fig. 27 - Block Diagram and Computer Set Up For Permutation
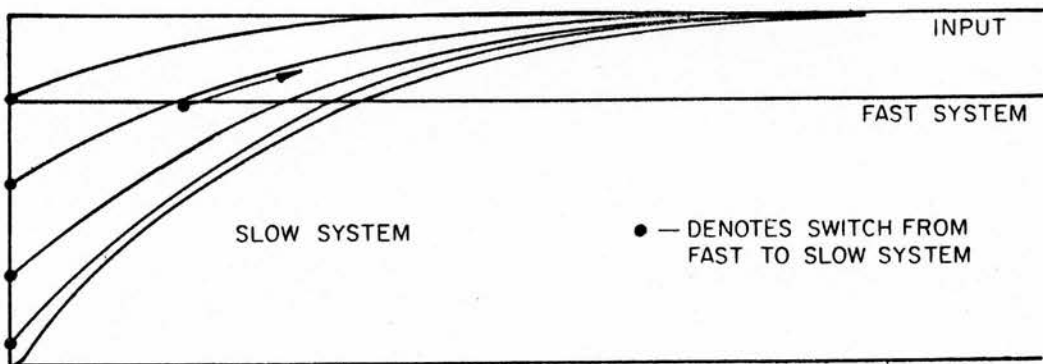
Fig. 28 - Responses for Permutation

These plots are characteristic of the responses obtained with the plants of 2 and 3 poles. Thus, the permutation is seen to yield compound results which plot as if mode 2 is merely responding to a step drive which is smaller than the actual input by the value that the switch occurs at. That this response does not follow the patterns established for the previous permutation is not disturbing.

Consider the following dual-mode system along with its permuted partner:

1st PERMUTATION   2nd PERMUTATION.

Fig. 29 - Block Diagram of Permutations

In both cases k_2 is taken $= 0$. The 1st permutation responds to the initial conditions present just before the switching takes place, while the 2nd immediately goes to zero (though the plant does not). It is also observed that the steady state plant behavior is radically different; the 1st permutation's plant going to zero and the 2nd's responding directly to a step input. The feedback further complicates the total non-linearity by insuring that the two errors (e(t))are different, magnifying the difference in the outputs.
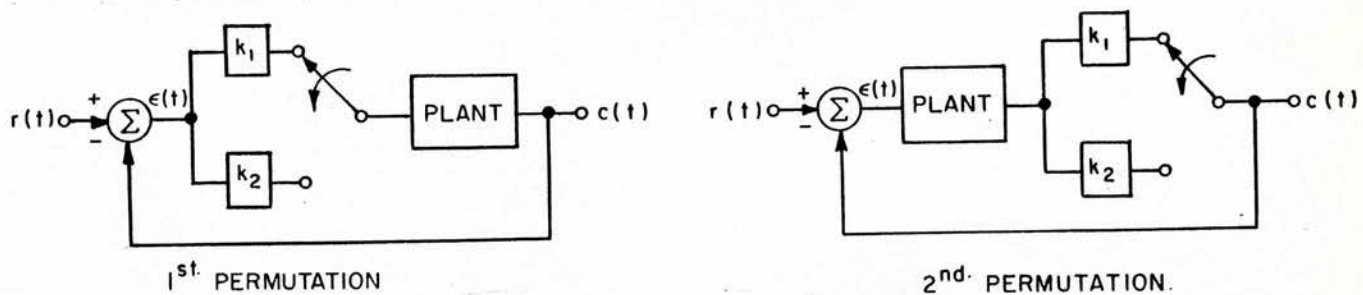
Mathematically the forward path is always subject to the following input-output relationships:

$$\phi (D) c(t) = k_1 \epsilon(t) \text{ before switching} \tag{21}$$

$$\phi (D) c(t) = k_2 \epsilon(t) \text{ after switching } (k_2 = 0) \tag{22}$$

$$\begin{cases} \phi (D) \text{ is a polynomial derivative operator gotten from} \\ \text{the denominator of the plant transfer function.} \end{cases}$$

Examining the second equation, the following possibilities arise:

1. c(t) is the homogeneous solution of the differential equation (taking into account the initial conditions).

2. c(t) and all its derivatives are immediately and always zero (this is the often forgotten trivial solution).

But which solution is correct ? This question cannot be answered without the use of the additional physical constraint of the circuit permutation. Obviously, possibility 1 corresponds to the physical constraint of the 1st permutation. Similarly permutation 2 is constraint enough to force the trivial solution (possibility 2).

Now allowing K_2 to be a non-zero constant and considering the same equation, these new possibilities are apparant:

1. The equation is solved in the ordinary way using the initial conditions prior to switching - allowing only the highest order deriative of c(t) to be discontinuous across the switch.

2. The instantaneous change of all derivative values of c(t) by a factor of $\frac{K_2}{K_1}$ and the solution of the differential equation using these new conditions.

Again only reference to an additional constraint, can indicate which solution is acceptable. Once this is done, possibilities 1 and 2 match permutations 1 and 2 respectively.

Thus, consideration of the dual mode system has shown that two entirely different solutions of the differential equation can be expected. Their applicability is dependent upon the physical requirements of the configuration. Changing K_2 to an integrator $\left(\dfrac{K_2}{S}\right)$ merely increases the order of $\phi(D)$:

$$\text{D.} \quad \phi(D)\, c(t) = K_2\, \varepsilon(t) \text{ after switching.}$$

Again, feedback further changes the output by modifying the input differently for such permutation.

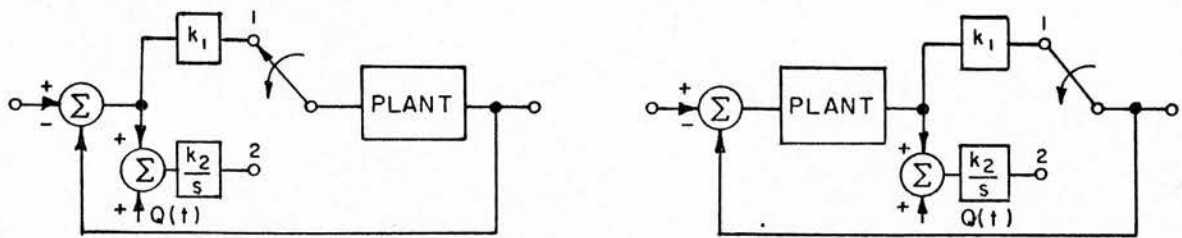Finally, to underscore this difference in overall results, the following two circuits are considered:

Fig. 30 - Completed Permutations

$Q(t)$ and $\underline{Q}(t)$ are impulses, doublets, triplets, etc. of proper magnitudes which would match all values and derivatives over the switch. It is fairly obvious that the results will still be divergent. This is so since the compensation can be considered one block whose transfer function is time variant (with no radical changes in velue or derivatives at the output of the block). It is a readily corroborated fact that a block diagram with time-dependent members cannot be permuted without disturbing the result.

V. A Consequent Technique:

A technique which holds much promise is the following. Substituting a ramp input for the step drive, the same compensation scheme as before is employed with any of the three previous plants. The object is to see if the overall system can follow the input. The motivation for this is that if a system can reasonably follow a randomly sloped ramp input, it will do well in following a completely arbitrary

input. Figure 11 is again taken to be the block diagram under consideration (with $v(t) = Kt$).

Placing the switch in position 1 yields a system whose output will cause the error to increase without bound for a ramp input. This is readily seen from an application of the final value theorem:

$$\varepsilon(t) = r(t)-c(t) \quad (23) \quad \text{and,} \quad \varepsilon(t) = \infty = \left\{ S\ R(s)\left[1 - T(s)\right]\right\}_{s\to 0} \quad (25) \quad ; \quad R(s) = K\left(\frac{1}{s^2}\right) \ (28)$$

$$= r(t)\left[1-\frac{c(t)}{r(t)}\right] \ (24)$$

$$= \left\{ \frac{K}{S}\left[\frac{(S+1)(S+10)^2}{(S+1)(S+10)^2 + K_1}\right]\right\}_{s\to 0} \quad (26)$$

$$= \infty \ . \quad (27)$$

Since the output is to follow the input, this increase in error is intolerable. Switching to position 2 and observing a typical error signal (steady state):

$$\varepsilon(t = \infty) = \left\{ s\ R(s)\left[1 - T(s)\right]\right\}_{s\to 0} \quad (29)$$

$$= \left\{ \frac{K}{S}\left[\frac{S(S+1)(S+10)^2}{S(S+1)(S+10)^2 + K_2}\right]\right\}_{s\to 0} \quad (30)$$

$$= \frac{100K}{K_2} \ . \quad (31)$$

Here it is seen that the error in steady state is finite. Thus, the output will tend to follow the input in steady state. The height of the output is always $\frac{100K}{K_2}$ less than the corresponding input.

Now, the technique consists of the following: The first system is designed to be unstable. The second system is designed to minimize the finite steady state error. The input is applied with the switch in position 1. The output begins to fly off to infinity (see Fig. 31).

However, before it can radically diverge from the input, the switch is set to position 2. The constraint of the second system is to bring the output (in steady state) to a value $\frac{100K}{K_2}$ less than the input. Even before this occurs, the switch is set back to position 1 and the process is again repeated. In this manner the compound output can be maintained oscillating about the input, well within the bounds of $\pm \frac{100K}{K_2}$. In fact it is readily seen that very narrow tolerance levels can easily be observed with a more sophisticated switching system and criterion.
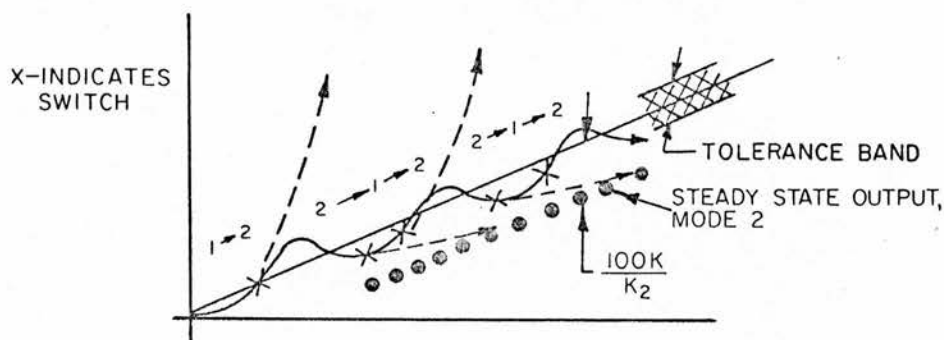
Fig. 31 Response to Technique

One of the interesting aspects of this problem, is to see if a switch operator can "learn" to control the behavior of the overall system. Thus, this problem can conceivably aid in an investigation of the learning process. In chemical processes, operators are known to be able to control complex processes which behave similar to the systems studied here, except that variations in the output occur over a matter of hours.

Unfortunately, stability problems, saturating amplifiers and the crude switching and measuring techniques used preclude a thorough investigation of this problem. For the case described, the difficulties above prevent the constraining of the output to a very narrow tolerance band, the transients (after each switch) far exceeding those limits. Time limits the restatement of the problem in more workable terms. It is mentioned here because of its extremely interesting aspects.

VI. Conclusion:

The employment of two simple compensation blocks and a switching arrangement has yielded a configuration whose outputs are very good approximations to the step inputs considered. Further, these results have been easily achieved using empirical techniques and have been shown to be independent of the plant chosen.

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